B.Tech. DEGREE EXAMINATION, MAY 2017
Third / Fourth Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015 – 2016 onwards)
(Statistical table to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. A random variable X has the following probability function

x:	0	1	2	3	4
P(x):	k	2k	5k	7k	9k

The value of K is.
(A) 2/24 (B) 21/24
(C) 7/12 (D) 1/24

2. $\text{var}[4X+8]$ is
(A) $12\,\text{var}[X]$ (B) $4\,\text{var}[X]+8$
(C) $16\,\text{var}[X]$ (D) $16\,\text{var}[X]+8$

3. When a die is thrown, X denotes the number that turns up, find the mean
(A) 7/2 (B) 3/4
(C) 4/5 (D) –4/5

4. Given that the PDF of a random variable X is $f(x)=2x;\ 0<x<1$, find $P(X>0.5)$
(A) 1/2 (B) 2/3
(C) 3/4 (D) 4/5

5. If on an average, 9 ships out of 10 arrive safely to a port, then mean and SD of the number of ships returning safely out of 150 ships are
(A) 135, 2.674 (B) 125, 3.674
(C) 135, 3.674 (D) 125, 2.674

6. Mean of the Poisson distribution is
(A) λ (B) $\lambda+1$
(C) $1/\lambda$ (D) λ^2

7. The MGF of Geometric distribution is
(A) $\dfrac{1}{1-qe^t}$ (B) $\dfrac{1}{1-pe^t}$
(C) $\dfrac{q}{1-pe^t}$ (D) $\dfrac{pe^t}{1-qe^t}$

8. The variance of Uniform distribution $U(a, b)$ is

(A) $\frac{1}{12}(b-a)^2$ (B) $\frac{1}{8}(b-a)^2$

(C) $\frac{1}{2}(b+a)$ (D) $\frac{1}{2}(b-a)$

9. The value set for α is known as

(A) Rejection level (B) Acceptance level

(C) Significance level (D) Hypothetical level

10. The standard deviation of a sampling distribution is called as

(A) Sampling error (B) Standard error

(C) Simple error (D) Sample error

11. A _____ is a subset of a _____

(A) Sample, population (B) Population, sample

(C) Statistic, parameter (D) Parameter, statistic

12. The hypothesis that an analyst is trying to prove is called

(A) Elective hypothesis (B) Alternate hypothesis

(C) Null hypothesis (D) Optional hypothesis

13. The symbolic notation of queuing model is represented by

(A) Kendall (B) Euler

(C) Fisher (D) Neuman

14. The interval between two consecutive arrivals of a Poisson process follows _____ distribution.

(A) Binomial (B) Uniform

(C) Normal (D) Exponential

15. The probability of 'n' customers in the system P_n = _____

(A) $\left(\frac{\lambda}{\mu}\right)P_0$ (B) $\left(\frac{\lambda}{\mu}\right)^n P_0$

(C) $\left(\frac{\mu}{\lambda}\right)^n P_0$ (D) $\left(\frac{\mu}{\lambda}\right)P_0$

16. Which term refers to "a customer who leaves the queue because the queue is too long"

(A) Balking (B) Reneging

(C) Jockeying (D) Leaving

17. The sum of all the elements of any row of the transition probability matrix is

(A) 0 (B) 0.5

(C) 0.75 (D) 1

18. The steady state probability vector π of a discrete markov chain with transition probability matrix satisfies the matrix equation.

(A) $\pi P = 0$ (B) $\pi P = \pi$

(C) $\pi(1-P) = 0$ (D) $\pi P = 1$

19. A non-null persistent and aperiodic state is called

(A) Empty (B) Finite

(C) Ergodic (D) Full

20. A state 'i' is said to be aperiodic with period 'di' if
(A) $di < 1$ (B) $di = 1$
(C) $di > 1$ (D) $di = 0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. A continuous random variable x has a pdf $f(x) = kx^2e^{-x}; x \geq 0$. Find the value of k.

22. Given the random variable x with density function
$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & elsewhere \end{cases}$. Find the pdf of $Y = 8X^3$.

23. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the fourth trial (ii) in fewer than 4 trials.

24. The mileage which can owner get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40, 000 km. Find the probabilities that one of these tires will last (i) atleast 20,000kms (ii) atmost 30, 000kms.

25. Define (i) null hypothesis (ii) alternate hypothesis (iii) type I error (iv) type II error.

26. What do the letters in the symbolic representation $(a/b/c):(d/e)$ of a queuing model represent?

27. If the tpm of a Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$. Find the steady state distribution of the chain.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution

x:	–2	–1	0	1	2	3
P(x):	0.1	k	0.2	2k	0.3	3k

(i) Find k (ii) evaluate $P(X < 2)$ (iii) $P(-2 < X < 2)$ (iv) CDF of X (v) mean of X (vi) variance of X.

(OR)

b.i If X is a random variable with $E(X)=3$ and $E(X^2)=13$, find the lower bound for $P(-2 < X < 8)$, using Tchebycheff's inequality.

ii. If X represents the outcome, when a fair die is tossed, find the MGF of X and hence find E(X) and var (X).

29. a. Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) atleast 1 boy (iii) atmost 2 girls (iv) children of both sexes. Assume equal probabilities for boys and girls.

(OR)

b.i. Buses arrive at a specified stop at 15 mins interval starting at 6 am (ie) arrive at 6am, 6.15 am, 6.30 am and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 6 and 6.30 am, find the probability that he waits (1) less than 5 mins for a bus (2) more than 10 mins for a bus.

ii. In a normal distribution, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?

30. a.i. In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?

ii. The mean height and the SD height of 8 randomly chosen soldiers are 166.9 and 8.29 cms respectively. The corresponding values of 6 randomly chosen sailors are 170.3 and 8.50 cm respectively. Based on these data, can we conclude that soldiers are, in general shorter than sailors?

(OR)

b. A total number of 3759 individuals were interviewed in a public opinion survey on a political proposal. Of them, 1872 were men and the rest women. A total of 2257 individuals were in favour of the proposal and 917 were opposed to it. A total of 243 men were undecided and 424 women were opposed to the proposal. Do you justify or contradict the hypothesis that there is no association between sex and attitude?

31. a. Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 12 min. Customers spend an average of 10 min in the barber's chair.
   (1) What is the expected number of customers in the barber shop and in the queue?
   (2) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.
   (3) How much time can a customer expect to spend in the barber's shop?
   (4) What is the average time customers spend in the queue?
   (5) What is the probability that the waiting time in the system is greater than 30 min?
   (6) Calculate the percentage of customers who have to wait prior to getting into the barber's chair.

(OR)

b. The local one-person barber shop can accommodate and maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 15 per hour. The barber cuts hair at an average rate of 4 per hour (exponential service time)
   (1) What percentage of time is the barber idle?
   (2) What fraction of the potential customers are turned away?
   (3) What is the expected number of customers waiting for a hair-cut?
   (4) How much time can a customer expect to spend in the barber shop?

32. a. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on the third day (ii) the probability that he drives to work in the long run.

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

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